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THE INFLUENCE OF DISTANT ZONES ON STOKES' EQUATION CONSIDERING --ETC(U)
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The expected error in the computation of geoid undulations using Stokes' equation due to neglected distant zones is analyzed considering the removal of lower-degree harmonics from limited gravity anomaly data and from Stokes' kernel. Evaluation of the derived error equations assuming Kaula's rule for anomaly degree variances indicates that if the lower-degree components of geoid undulation are assumed known, removal of an equivalent number of such harmonics from the anomaly data alone produces the least-expected error for cap radii of less than 60 deg.

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FOREWORD

The work reported herein was conducted by the Ocean Geodesy Branch of the Naval Surface Weapons Center under the sponsorship of the Strategic Systems Project Office (Project Number J0094).

This report has been reviewed and approved by P. Ugincius, Head, Ocean Geodesy Branch and R. J. Anderle, Head, Astronautics and Geodesy Division.

Released by

R. A. NIEMANN, Head
Strategic Systems Department

CONTENTS

INTRODUCTION

Stokes' equation gives the mathematical relationship between the geometric shape of the geoid and the magnitude of gravity on the geoid. The equation is

$$N = \frac{R}{4\pi G}\iint S(\psi)\, \Delta g \, d\sigma \tag{1}$$

where

R = mean radius of the earth

G = mean value of gravity over the earth

$S(\psi)$ = Stokes' kernel

The integration, extended over the unit sphere using a global set of gravity anomalies Δg, produces the geoid-ellipsoid separation at a given point.

In practice, the integration is not extended over the entire globe, but only up to a spherical distance ψ_0 from the computation point due to a lack of sufficient surface gravity data. The error δN that results from neglecting data beyond ψ_0 has been analyzed by Molodenskii (Reference 1) and is described in Moritz (Reference 2). The magnitude of this error may be reduced if the gravitational potential is known in terms of a finite spherical harmonic expansion derived from satellite observations by applying, for instance, the computational technique of Rummel and Rapp (Reference 3). Equivalently, the error may be reduced by removing known lower-degree harmonies from the gravity anomaly data used when Equation (1) is applied to a spherical cap assuming the lower-order harmonic components of N are known. In addition, one might also consider the possibility of modifying or replacing Stokes' kernel $S(\psi)$ with a function which produces a better estimate of N using a limited data set. The purpose of this report is to determine what decrease in δN may be expected if harmonics are removed from $S(\psi)$ or Δg individually, or from both simultaneously.

INFLUENCE OF DISTANT ZONES ON STOKES' EQUATION

CASE I: STANDARD CASE

The error in the geoid undulation due to neglecting gravity anonaly data outside a spherical cap σ of radius ψ_0 is given by Moritz (Reference 2) as

$$\delta N_I = \frac{R}{4\pi G}\int_{\psi=\psi_0}^{\pi}\int_{\alpha=0}^{2\pi} S(\psi)\, \Delta g \sin\psi \, d\psi \, d\alpha \tag{2}$$

where the subscript I refers to the removal of harmonics from the data only (Table 1). Defining a new function $\bar{S}(\psi)$ as

$$\overline{S}(\psi) = \begin{cases} 0 & 0 \leq \psi < \psi_0 \\ S(\psi) & \psi_0 \leq \psi \leq \pi \end{cases} \tag{3}$$

Equation (2) becomes

$$\delta N_I = \frac{R}{4\pi G} \int_{\psi=0}^{\pi} \int_{\alpha=0}^{2\pi} \overline{S}(\psi)\, \Delta g \sin\psi \, d\psi \, d\alpha \tag{4}$$

The function $\overline{S}(\psi)$ has the following spherical harmonic expansion:

$$\overline{S}(\psi) = \sum_{n=0}^{\infty} \frac{2n+1}{2} Q_n P_n(\cos\psi) \tag{5}$$

where

$$Q_n = \int_0^{\pi} \overline{S}(\psi) P_n(\cos\psi) \sin\psi \, d\psi$$

or (6)

$$= \int_{\psi_0}^{\pi} S(\psi) P_n(\cos\psi) \sin\psi \, d\psi$$

Following Moritz (Reference 2), the RMS error due to neglecting distant zones is the square root of

$$\overline{\delta N^2} = \frac{R^2}{4G^2} \sum_{n=2}^{\infty} Q_n^2 c_n \tag{7}$$

where it is assumed the zero and first-order harmonics of the gravity anomaly field are zero.

$$\Delta g(\theta,\lambda) = \sum_{n=2}^{\infty} \Delta g_n(\theta,\lambda) \tag{8}$$

The quantity c_n is the anomaly degree variance for degree n defined by

$$c_n = E\left\{\Delta g_n^2\right\} \tag{9}$$

where E is the expected value operator. The anomaly degree variance is a function of the spherical harmonic coefficients of degree n

$$c_n = \sum_{m=0}^{n} (\overline{a}_{nm}^2 + \overline{b}_{nm}^2) \tag{10}$$

where the bar indicates normalized coefficients. The coefficients Q_n are known as the Molodenskii coefficients, and much is known of their properties.

CASE II: REMOVAL OF M LOWER-DEGREE HARMONICS FROM S(ψ)

Equation (1) may be written in the following form:

$$N = \frac{R}{4\pi G} \int_{\psi=0}^{\psi_0} \int_{\alpha=0}^{2\pi} S(\psi)\, \Delta g \sin\psi \, d\psi \, d\alpha$$

$$+ \frac{R}{4\pi G} \int_{\psi=\psi_0}^{\pi} \int_{\alpha=0}^{2\pi} S(\psi)\, \Delta g \sin\psi \, d\psi \, d\alpha \tag{11}$$

The kernel S(ψ) has the spherical harmonic expansion

$$S(\psi) = \sum_{n=2}^{\infty} \frac{2n+1}{n-1} P_n(\cos\psi) \tag{12}$$

Now define

$$\tilde{S}(\psi) = S(\psi) - \sum_{n=2}^{m} \frac{2n+1}{n-1} P_n(\cos\psi) \tag{13a}$$

or

$$S(\psi) = \tilde{S}(\psi) + S_m(\psi) \tag{13b}$$

Substituting Equation (13b) into Equation (11) gives

$$N = I_1 + I_2 + I_3 \tag{14a}$$

where

$$I_1 = \frac{R}{4\pi G} \int_{\psi=0}^{\psi_0} \int_{\alpha=0}^{2\pi} \tilde{S}(\psi)\, \Delta g \, d\sigma \tag{14b}$$

$$I_2 = \frac{R}{4\pi G} \int_{\psi=\psi_0}^{\pi} \int_{\alpha=0}^{2\pi} \tilde{S}(\psi)\, \Delta g \, d\sigma \tag{14c}$$

$$I_3 = \frac{R}{4\pi G} \int_{\psi=0}^{\pi} \int_{\alpha=0}^{2\pi} S_m(\psi)\, \Delta g \, d\sigma \tag{14d}$$

Considering the definition of $S_m(\psi)$ and Equation (8), the integral I_3 becomes

$$I_3 = N_2 + N_3 + \ldots + N_m \tag{15}$$

which are the first m harmonic components of N at the computation point. Assuming I_3 to be known, the error due to integration over a limited cap becomes

$$\delta N_{II} = \frac{R}{4\pi G} \int_{\psi=\psi_0}^{\pi} \int_{\alpha=0}^{2\pi} \tilde{S}(\psi)\Delta g \, d\sigma \tag{16}$$

Define the function $\tilde{\bar{S}}$ as

$$\tilde{\bar{S}}(\psi) = \begin{cases} 0 & 0 \leqslant \psi < \psi_0 \\ \tilde{S}(\psi) & \psi_0 \leqslant \psi \leqslant \pi \end{cases}$$

$$= \begin{cases} 0 & 0 \leqslant \psi < \psi_0 \\ S(\psi) - S_m(\psi) & \psi_0 \leqslant \psi \leqslant \pi \end{cases} \tag{17}$$

The function $\tilde{\bar{S}}$ has the Legendre series expansion

$$\tilde{\bar{S}}(\psi) = \sum_{n=0}^{\infty} \frac{2n+1}{2} \tilde{Q}_n P_n(\cos\psi) \tag{18}$$

where

$$\begin{aligned} \tilde{Q}_n &= \int_0^{\pi} \tilde{\bar{S}}(\psi) P_n(\cos\psi) \sin\psi \, d\psi \\ &= \int_{\psi_0}^{\pi} S(\psi) P_n(\cos\psi) \sin\psi \, d\psi - \int_{\psi_0}^{\pi} S_m(\psi) P_n(\cos\psi) \sin\psi \, d\psi \\ &= Q_n - \int_{\psi_0}^{\pi} S_m(\psi) p_n(\cos\psi) \sin\psi \, d\psi \end{aligned} \tag{19}$$

Therefore,

$$\begin{aligned} \delta N_{II} &= \frac{R}{4\pi G} \int_{\psi_0}^{\pi} \int_0^{2\pi} \tilde{S}(\psi) \Delta g \, d\sigma \\ &= \frac{R}{4\pi G} \int_0^{\pi} \int_0^{2\pi} \tilde{\bar{S}}(\psi) \Delta g \, d\sigma \end{aligned}$$

$$= \frac{R}{8\pi G} \sum_{n=0}^{\infty} (2n + 1) \tilde{Q}_n \int_0^{\pi} \int_0^{2\pi} \Delta g \, P_n(\cos \psi) \sin \psi \, d\psi \, d\alpha$$

$$= \frac{R}{8\pi G} \sum_{n=0}^{\infty} (2n + 1) \tilde{Q}_n \frac{4\pi \, \Delta g_n}{(2n + 1)}$$

$$= \frac{R}{2G} \sum_{n=0}^{\infty} \tilde{Q}_n \, \Delta g_n \tag{20}$$

The RMS error is, following Moritz (Reference 2),

$$\overline{\delta N_{II}^2} = \frac{R^2}{4G^2} \sum_{n=2}^{\infty} \tilde{Q}_n^2 \, c_n \tag{21}$$

CASE III: REMOVAL OF M LOWER-DEGREE HARMONICS FROM Δg

From Equation (8),

$$\Delta g = \sum_{n=2}^{\infty} \Delta g_n(\theta,\lambda)$$

where the n'th degree hamonic Δg_n is given by

$$\Delta g_n = \sum_{m=0}^{n} (\bar{a}_{nm} \overline{R}_{nm} (\theta,\lambda) + \overline{b}_{nm} \overline{S}_{nm} (\theta,\lambda)) \tag{22}$$

Removing the first m harmonics from the data set gives

$$\widetilde{\Delta g} = \Delta g - \sum_{n=2}^{m} \Delta g_n \tag{23}$$

Equation (1) becomes

$$N = \frac{R}{4\pi G} \int_0^{\pi} \int_0^{2\pi} S(\psi) \, [\widetilde{\Delta g} + \sum_{n=2}^{m} \Delta g_n] \sin \psi \, d\psi \, d\alpha$$

$$= I_1 + I_2 \tag{24a}$$

where

$$I_1 = \frac{R}{4\pi G}\int_0^{\pi}\int_0^{2\pi} S(\psi)\,\widetilde{\Delta g}\,\sin\psi\,d\psi\,d\alpha \tag{24b}$$

$$I_2 = \frac{R}{4\pi G}\sum_{n=2}^{m}\int_0^{\pi}\int_0^{2\pi} S(\psi)\,\Delta g_n\,\sin\psi\,d\psi\,d\alpha \tag{24c}$$

The second integral reduces to

$$I_2 = N_2 + \ldots + N_m \tag{25}$$

which is assumed known since $\Delta g_2, \ldots \Delta g_m$ are assumed given. Therefore,

$$\begin{aligned} N = N_2 + \ldots + N_m &+ \frac{R}{4\pi G}\int_0^{\psi_0}\int_0^{2\pi} S(\psi)\,\widetilde{\Delta g}\,d\sigma \\ &+ \frac{R}{4\pi G}\int_0^{\pi}\int_0^{2\pi} S(\psi)\,\widetilde{\Delta g}\,d\sigma \end{aligned} \tag{26}$$

The error due to distant zones is the second integral in Equation (26).

$$\begin{aligned} \delta N_{III} &= \frac{R}{4\pi G}\int_{\psi_0}^{\pi}\int_0^{2\pi} S(\psi)\,\widetilde{\Delta g}\,d\sigma \\ &= \frac{R}{4\pi G}\int_0^{\pi}\int_0^{2\pi} \overline{S}(\psi)\,\widetilde{\Delta g}\,d\sigma \\ &= \frac{R}{8\pi G}\sum_{n=0}^{\infty}(2n+1)\,Q_n\int_0^{\pi}\int_0^{2\pi} \widetilde{\Delta g}\,P_n(\cos\psi)\sin\psi\,d\psi\,d\alpha \end{aligned} \tag{27}$$

For n = 0, 1, . . . , m

$$\int_0^{\pi}\int_0^{2\pi} \widetilde{\Delta g}\,P_n(\cos\psi)\sin\psi\,d\psi\,d\alpha = 0 \tag{28}$$

since $\widetilde{\Delta g}$ contains no harmonics of these degrees. Therefore,

$$\delta N_{III} = \frac{R}{2G} \sum_{n=m+1}^{\infty} Q_n \, \Delta g_n \tag{29}$$

and

$$\overline{\delta N_{III}^2} = \frac{R^2}{4G^2} \sum_{n=m+1}^{\infty} Q_n^2 \, c_n \tag{30}$$

It should be noted that this error is the same as that obtained using the Rummel and Rapp (Reference 3) procedure with the first m harmonics applied (neglecting here corrections for atmospheric effects and zero-order undulation correction).

CASE IV: REMOVAL OF M LOWER-DEGREE HARMONICS FROM BOTH S(ψ) AND Δg

Using equations (13b) and (23),

$$S = \tilde{S} + S_m \tag{13b}$$

$$\Delta g = \widetilde{\Delta g} + \sum_{n=2}^{m} \Delta g_n \tag{23}$$

Equation (1) becomes

$$N = I_1 + I_2 + I_3 + I_4 \tag{31a}$$

where

$$I_1 = \frac{R}{4\pi G} \int_0^{\pi} \int_0^{2\pi} \tilde{S}(\psi) \, \Delta g \, d\sigma \tag{31b}$$

$$I_2 = \frac{R}{4\pi G} \sum_{n=2}^{m} \int_0^{\pi} \int_0^{2\pi} \tilde{S}(\psi) \, \Delta g_n \, d\sigma \tag{31c}$$

$$I_3 = \frac{R}{4\pi G} \int_0^{\pi} \int_0^{2\pi} S_m(\psi) \, \widetilde{\Delta g} \, d\sigma \tag{31d}$$

$$I_4 = \frac{R}{4\pi G} \sum_{n=2}^{m} \int_0^{\pi} \int_0^{2\pi} S_m(\psi)\, \Delta g_n \, d\sigma \tag{31e}$$

By orthogonality, I_2 and I_3 vanish. Also, by definition of S_m (Equation 13a),

$$I_4 = \frac{R}{4\pi G} \sum_{n=2}^{m} \sum_{j=2}^{m} \frac{2j+1}{j-1} \int_0^{\pi} \int_0^{2\pi} P_j(\cos\psi)\, \Delta g_n \, d\sigma$$

If $n \neq j$, the partial sum is zero by orthogonality. When $n = j$, the nonvanishing terms sum to

$$I_4 = \frac{R}{4\pi G} \sum_{n=2}^{m} \frac{2n+1}{n-1} \Delta g_n \frac{4\pi}{2n+1} \tag{32}$$

since

$$\int_0^{\pi} \int_0^{2\pi} P_n(\cos\psi)\, \Delta g_n \, d\sigma = \frac{4\pi\, \Delta g_n}{2n+1}$$

Therefore,

$$I_4 = N_2 + \ldots + N_m \tag{33}$$

Thus, Equation (31a) becomes

$$N = N_2 + \ldots + N_m + \frac{R}{4\pi G} \int_{\psi=0}^{\psi_0} \int_{\alpha=0}^{2\pi} \widetilde{S}(\psi)\, \widetilde{\Delta g}\, d\sigma$$

$$+ \frac{R}{4\pi G} \int_{\psi=\psi_0}^{\pi} \int_{\alpha=0}^{2\pi} \widetilde{S}(\psi)\, \widetilde{\Delta g}\, d\sigma \tag{34}$$

Assuming the first m harmonics of N are known, the error in N due to neglecting distant zones is

$$\delta N_{IV} = \frac{R}{4\pi G} \int_{\psi_0}^{\pi} \int_0^{2\pi} \widetilde{S}(\psi)\, \widetilde{\Delta g}\, d\sigma$$

$$= \frac{R}{4\pi G} \int_0^{\pi} \int_0^{2\pi} \tilde{\tilde{S}}(\psi)\, \tilde{\Delta g}\, d\sigma$$

$$= \frac{R}{8\pi G} \sum_{n=0}^{\infty} (2n+1)\, \tilde{Q}_n \int_{\psi=0}^{\pi} \int_{\alpha=0}^{2\pi} \tilde{\Delta g}\, P_n(\cos\psi)\, d\sigma \tag{35}$$

For n = 0, 1, . . . , m the integral in Equation (35) vanishes giving

$$\delta N_{IV} = \frac{R}{2G} \sum_{n=m+1}^{\infty} \tilde{Q}_n\, \Delta g_n \tag{36}$$

The RMS error is the square root of

$$\overline{\delta N_{IV}^2} = \frac{R^2}{4G^2} \sum_{n=m+1}^{\infty} \tilde{Q}_n^2\, c_n \tag{37}$$

Tables 1 and 2 present summaries of Cases I through IV and of RMS error formulae, respectively.

Table 1. Summary of Cases I through IV

Case	Equation for Undulation
I	$N = \frac{R}{4\pi G} \iint_\sigma S(\psi)\, \Delta g\, d\sigma + \delta N_I$
II	$N = N_2 + \ldots + N_m + \frac{R}{4\pi G} \iint_\sigma \tilde{S}(\psi)\, \Delta g\, d\sigma + \delta N_{II}$
III	$N = N_2 + \ldots + N_m + \frac{R}{4\pi G} \iint_\sigma S(\psi)\, \tilde{\Delta g}\, d\sigma + \delta N_{III}$
IV	$N = N_2 + \ldots + N_m + \frac{R}{4\pi G} \iint_\sigma \tilde{S}(\psi)\, \tilde{\Delta g}\, d\sigma + \delta N_{IV}$

Table 2. Summary of RMS Error Formulae

Case	M Harmonics Removed	Square of RMS Error
I	–	$\frac{R^2}{4G^2}\sum_{n=2}^{\infty} Q_n^2 c_n$
II	$S(\psi)$	$\frac{R^2}{4G^2}\sum_{n=2}^{\infty} \tilde{Q}_n^2 c_n$
III	Δg	$\frac{R^2}{4G^2}\sum_{n=m+1}^{\infty} Q_n^2 c_n$
IV	$S(\psi)$, Δg	$\frac{R^2}{4G^2}\sum_{n=m+1}^{\infty} \tilde{Q}_n^2 c_n$

NUMERICAL EVALUATION OF ERROR FORMULAE

The RMS error for each case was computed for m equal to 6, 12, and 18. In these computations the anomaly degree variances were computed using Kaula's rule

$$c_n = \frac{192}{n + 1.5} \text{ mgals}^2 \quad n = 3, 4, \ldots \tag{38}$$

and c_2 was taken as 10 mgal^2. The constant R^2/G^2 was taken as 42.3 m^2/mgal^2.

The Molodenskii (Reference 1) coefficients Q_n were computed using two algorithms. Initially, the Q_n were computed using an algorithm of Hagiwara (Reference 4); however, due to numerical difficulties, only the first 50 terms in the error expressions could be summed. The computations were repeated using the algorithm of Paul (Reference 5). An increased number of terms (250) were summed in this case.

The coefficients $\tilde{Q}_n$ were computed using the following procedure:

$$\tilde{Q}_n = Q_n - \int_{\psi_0}^{\pi} S_m P_n (\cos \psi) \sin \psi \, d\psi \tag{39}$$

$$S_m = \sum_{j=2}^{m} \frac{2j + 1}{j - 1} P_j(\cos \psi) \tag{40}$$

$$\tilde{Q}_n = Q_n - \sum_{j=2}^{m} \frac{2j+1}{j-1} \int_{\psi_0}^{\pi} P_j(\cos\psi)\, P_n(\cos\psi) \sin\psi \, d\psi \tag{41}$$

where $\quad Q_n = Q_n(\psi_0)$

$\qquad\quad \tilde{Q}_n = \tilde{Q}_n(\psi_0)$

Taking $\quad x = \cos\psi$

$$\int_{\psi_0}^{\pi} P_j(\cos\psi)\, P_n(\cos\psi) \sin\psi \, d\psi$$

$$= \int_{-1}^{\cos\psi_0} P_j(x)\, P_n(x)\, dx, \tag{42}$$

and using the identities

$$\int_a^b P_n^2(x)\, dx = \frac{1}{2n+1} \left\{ (2n-1) \int_a^b P_{n-1}^2(x)\, dx + x\,[P_n^2(x) + P_{n-1}^2(x)] - 2P_{n-1} P_n \right\} \Bigg|_a^b \tag{43}$$

$$\int_a^b P_n(x)\, P_k(x)\, dx = \frac{[x(n-k)\, P_n(x)\, P_k(x) + kP_n(x)\, P_{k-1}(x) - nP_{n-1}(x)\, P_k(x)]}{(n-k)(n+k+1)} \Bigg|_a^b$$

$$n \neq k \tag{44}$$

Equation (41) may be evaluated.

CONCLUSIONS

Figures 1 through 4 give the root mean square error for Cases I through IV where 250 terms have been summed in the error equations. In Cases II through IV, the first m harmonic components of the geoid undulation N were assumed known. An examination of these figures shows that for spherical caps of radii $\psi_0 < 60°$, removal of known lower-degree harmonics from the cap data alone produces the least-expected error due to neglected zones. Removal of lower-degree harmonic components from the kernel function $S(\psi)$ does not minimize the expected error for cap sizes normally considered. Thus, the former procedure (Case III) is recommended if the assumptions, upon which these error formulae are based, are satisfied.

Finally, a comparison of the results based on 50 and 250 terms in the summation demonstrated that satisfactory convergence of the series had been obtained.

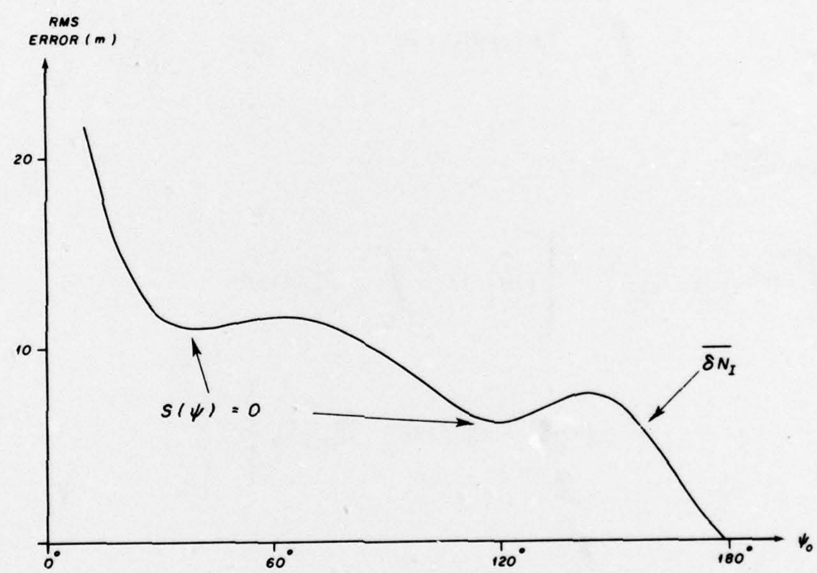

Figure 1. Root Mean Square Error, Case I

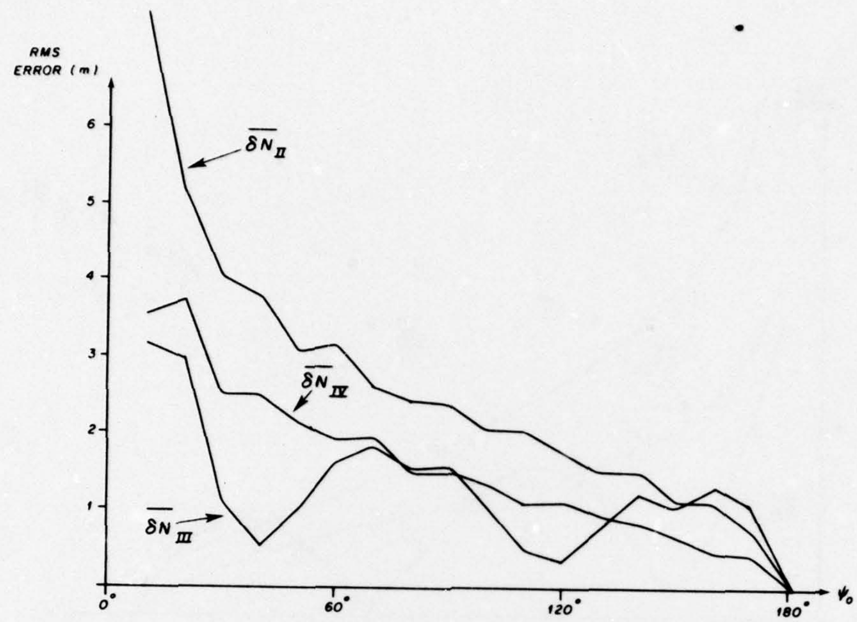

Figure 2. Root Mean Square Error,
Cases II Through IV, m = 6

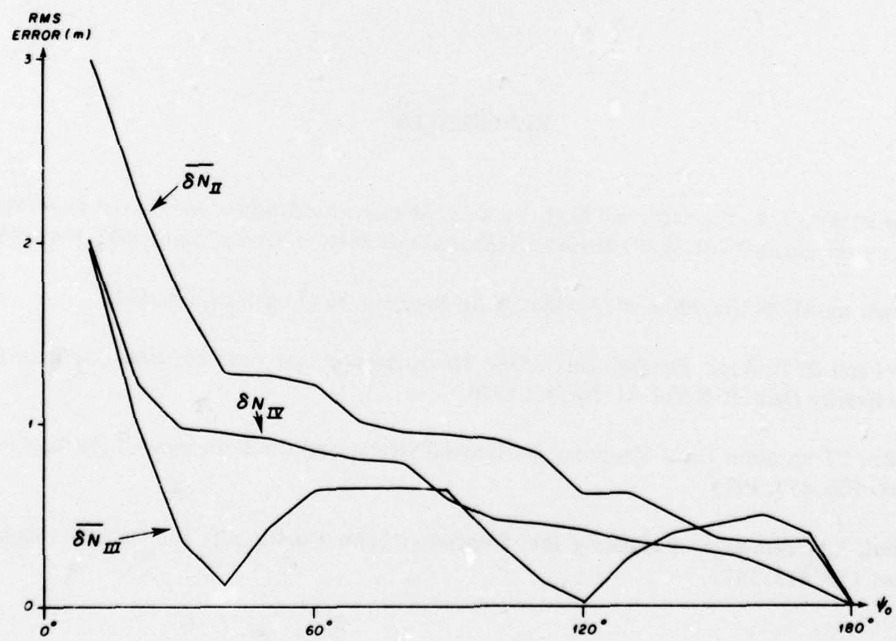

Figure 3. Root Mean Square Error,
Cases II Through IV, m = 12

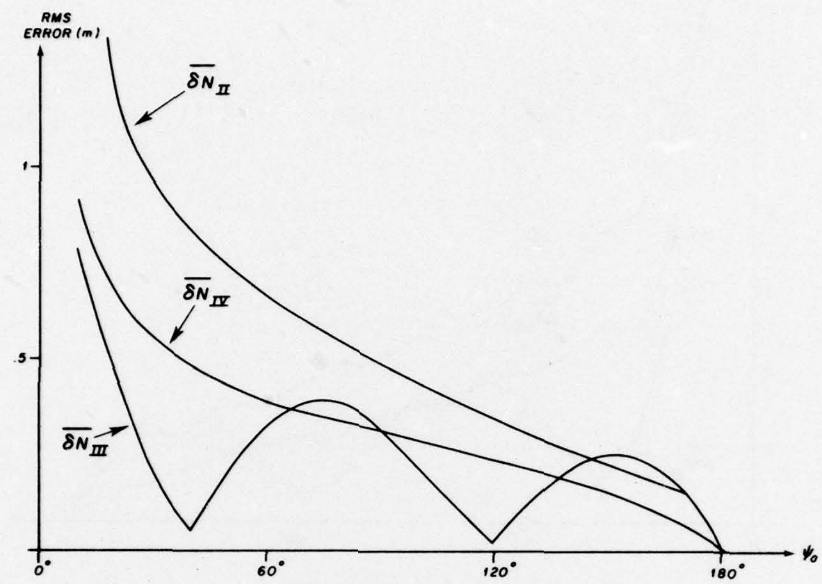

Figure 4. Root Mean Square Error,
Cases II through IV, m = 18

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The Ohio State University
Department of Geodetic Science
164 W. Nineteenth Avenue
Columbus, OH 43210
Attn: Dr. I. I. Mueller
Dr. U. A. Uotila
Dr. R. Rapp

Aerospace Corporation
2350 East El Segundo Boulevard
El Segundo, CA
Attn: R. Farrar
L. Wong

Department of Civil Engineering
Purdue University
Lafayette, IN
Attn: Prof. L. Kivioja

Division of National Mapping
P.O. Box 667
Canberra City, A.C.T. 2601
Australia
Attn: J. Luck

Technishe Hochschule Graz
Technikerstr. 4, A-8010 Graz
Austria
Attn: Prof. P. Meissl
Prof. H. Moritz
Prof. K. Rinner

Universidade Federal Do Parana
Curso De Pos
Graduacao Em Ciencias Geodesicas
CX. P. 1611
60.000, Curitiba, Parana, Brazil
Attn: Prof. C. Gemael
Prof. J. Bittencourt de Andrade

Geodetic Survey of Canada
615 Booth St., Ottawa, Canada
Attn: J. Kolba
A. Peterson
A. Porter

Department of Survey Engineering
University of New Brunswick
Fredericton, N.B., Canada
Attn: Dr. P. Vanicek

Geodetic Institute of the Delft
Technological University
Delft, Netherlands
Attn: Dr. J. E. Alberda
Dr. L. Aardoy
Dr. B. Van Gelder

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